Dynamic Models, Autocorrelation and Forecasting

Chapter 9

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Chapter 9: Dynamic Models, Autocorrelation and Forecasting

9.1 Introduction

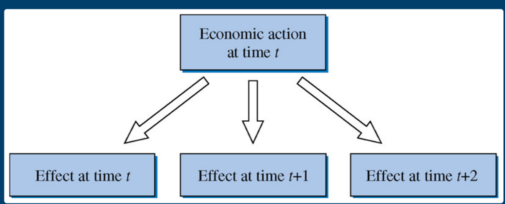

Figure 9.1

9.1 Introduction

$$y_t = f(x_t, x_{t-1}, x_{t-2}, \ldots) \quad (9.1)$$

$$y_t = f(y_{t-1}, x_t) \quad (9.2)$$

$$y_t = f(x_t) + e_t \qquad e_t = f(e_{t-1}) \quad (9.3)$$

9.1 Introduction

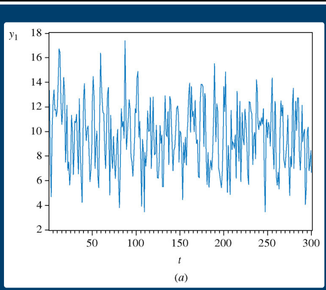

Figure 9.2(a) Time Series of a Stationary Variable

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9.1 Introduction

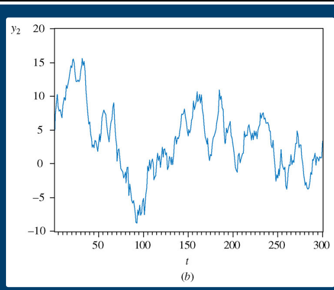

Figure 9.2(b) Time Series of a Nonstationary Variable that is 'Slow Turning' or 'Wandering'

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9.1 Introduction

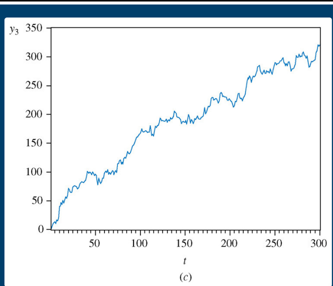

Figure 9.2(c) Time Series of a Nonstationary Variable that 'Trends'

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9.2 Lags in the Error Term: Autocorrelation

9.2.1 Area Response Model for Sugar Cane

$$\ln(A) = \beta_1 + \beta_2 \ln(P)$$

$$\ln(A_t) = \beta_1 + \beta_2 \ln(P_t) + e_t \quad (9.4)$$

$$y_t = \beta_1 + \beta_2 x_t + e_t \quad (9.5)$$

$$e_t = \rho e_{t-1} + v_t \quad (9.6)$$

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9.2.2 First-Order Autoregressive Errors

$$y_t = \beta_1 + \beta_2 x_t + e_t \quad (9.7)$$

$$e_t = \rho e_{t-1} + v_t \quad (9.8)$$

$$E(v_t) = 0 \quad \text{var}(v_t) = \sigma_v^2 \quad \text{cov}(v_t, v_s) = 0 \quad \text{for } t \neq s \quad (9.9)$$

$$-1 < \rho < 1 \quad (9.10)$$

9.2.2 First-Order Autoregressive Errors

$$E(e_t) = 0 \quad (9.11)$$

$$\text{var}(e_t) = \sigma_e^2 = \frac{\sigma_v^2}{1-\rho^2} \quad (9.12)$$

$$\text{cov}\left(e_t, e_{t-k}\right) = \sigma_e^2 \rho^k \qquad k > 0 \quad (9.13)$$

9.2.2 First-Order Autoregressive Errors

$$\text{corr}(e_t, e_{t-k}) = \frac{\text{cov}(e_t, e_{t-k})}{\sqrt{\text{var}(e_t)\,\text{var}\left(e_{t-k}\right)}} = \frac{\text{cov}(e_t, e_{t-k})}{\text{var}(e_t)} = \frac{\sigma_e^2 \rho^k}{\sigma_e^2} = \rho^k \quad (9.14)$$

$$\text{corr}(e_t, e_{t-1}) = \rho \quad (9.15)$$

$$\hat{y}_t = 3.893 + .776\,x_t$$
$$(\text{se}) \quad (.061) \quad (.277) \quad (9.16)$$

9.2.2 First-Order Autoregressive Errors

Table 9.1 Least Squares Residuals for the Sugarcane Example

Time	$\hat{e}_t$	Time	$\hat{e}_t$	Time	$\hat{e}_t$	Time	$\hat{e}_t$
1	−0.303	10	−0.254	19	−0.036	27	−0.651
2	0.254	11	−0.145	20	0.361	28	−0.218
3	0.182	12	0.091	21	−0.138	29	0.137
4	0.503	13	0.304	22	0.017	30	0.121
5	0.275	14	0.656	23	0.336	31	−0.040
6	−0.115	15	0.134	24	−0.175	32	−0.048
7	−0.437	16	−0.059	25	−0.517	33	0.183
8	−0.423	17	0.435	26	−0.137	34	0.184
9	−0.367	18	−0.106				

9.2.2 First-Order Autoregressive Errors

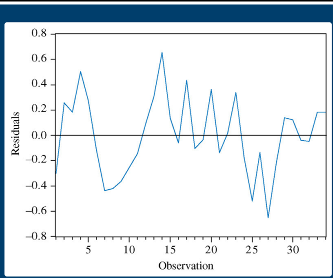

Figure 9.3 Least Squares Residuals Plotted Against Time

9.2.2 First-Order Autoregressive Errors

$$r_{xy} = \frac{\widehat{\text{cov}}(x_t, y_t)}{\sqrt{\widehat{\text{var}}(x_t)\widehat{\text{var}}(y_t)}} = \frac{\sum_{t=1}^{T}(x_t - \bar{x})(y_t - \bar{y})}{\sqrt{\sum_{t=1}^{T}(x_t - \bar{x})^2 \sum_{t=1}^{T}(y_t - \bar{y})^2}} \quad (9.17)$$

$$r_1 = \frac{\widehat{\text{cov}}(e_t, e_{t-1})}{\sqrt{\widehat{\text{var}}(e_t)}} = \frac{\sum_{t=2}^{T}\hat{e}_t\hat{e}_{t-1}}{\sum_{t=2}^{T}\hat{e}_{t-1}^2} \quad (9.18)$$

9.3 Estimating an AR(1) Error Model

The existence of AR(1) errors implies:

- The least squares estimator is still a linear and unbiased estimator, but it is no longer best. There is another estimator with a smaller variance.
- The standard errors usually computed for the least squares estimator are incorrect. Confidence intervals and hypothesis tests that use these standard errors may be misleading.

9.3 Estimating an AR(1) Error Model

Sugar cane example

The two sets of standard errors, along with the estimated equation are:

$$\hat{y}_t = 3.893 + .776x_t$$

(.061)	(.277)	'incorrect' se's
(.062)	(.378)	'correct' se's

The 95% confidence intervals for β_2 are:

(.211, 1.340) (incorrect)

(.006, 1.546) (correct)

9.3.2 Nonlinear Least Squares Estimation

$$y_t = \beta_1 + \beta_2 x_t + e_t \quad (9.19)$$

$$e_t = \rho e_{t-1} + v_t \quad (9.20)$$

$$y_t = \beta_1 + \beta_2 x_t + \rho e_{t-1} + v_t \quad (9.21)$$

$$e_{t-1} = y_{t-1} - \beta_1 - \beta_2 x_{t-1} \quad (9.22)$$

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9.3.2 Nonlinear Least Squares Estimation

$$\rho e_{t-1} = \rho y_{t-1} - \rho\beta_1 - \rho\beta_2 x_{t-1} \quad (9.23)$$

$$y_t = \beta_1(1-\rho) + \beta_2 x_t + \rho y_{t-1} - \rho\beta_2 x_{t-1} + v_t \quad (9.24)$$

$$\begin{array}{lll} \ln(A_t) = 3.899 + .888\ln(P_t) & & e_t = .422 e_{t-1} + v_t \\ \text{(se)} \quad (.092) \quad (.259) & & \quad (.166) \end{array} \quad (9.25)$$

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9.3.2a Generalized Least Squares Estimation

It can be shown that nonlinear least squares estimation of (9.24) is equivalent to using an iterative generalized least squares estimator called the Cochrane-Orcutt procedure. Details are provided in Appendix 9A.

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9.3.3 Estimating a More General Model

$$y_t = \beta_1(1-\rho) + \beta_2 x_t - \rho\beta_2 x_{t-1} + \rho y_{t-1} + v_t \quad (9.26)$$

$$y_t = \delta + \delta_0 x_t + \delta_1 x_{t-1} + \theta_1 y_{t-1} + v_t \quad (9.27)$$

$$\delta = \beta_1(1-\rho) \qquad \delta_0 = \beta_2 \qquad \delta_1 = -\rho\beta_2 \qquad \theta_1 = \rho$$

$$\begin{array}{l} \hat{y}_t = 2.366 + .777 x_t - .611 x_{t-1} + .404 y_{t-1} \\ \text{(se)} \quad (.656) \quad (.280) \quad (.297) \quad (.167) \end{array} \quad (9.28)$$

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9.4 Testing for Autocorrelation

9.4.1 Residual Correlogram

$$H_0 : \rho = 0 \qquad H_1 : \rho \neq 0$$

$$z = \sqrt{T}\, r_1 \sim N(0,1) \quad (9.29)$$

$$z = \sqrt{34} \times .404 = 2.36 \geq 1.96 \quad (9.30)$$

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9.4 Testing for Autocorrelation

9.4.1 Residual Correlogram

$$r_1 \geq \frac{1.96}{\sqrt{T}} \quad \text{or} \quad r_1 \leq -\frac{1.96}{\sqrt{T}}$$

$$r_k \geq \frac{1.96}{\sqrt{T}} \quad \text{or} \quad r_k \leq -\frac{1.96}{\sqrt{T}} \quad (9.31)$$

$$\rho_k = \frac{\mathrm{cov}(e_t, e_{t-k})}{\mathrm{var}(e_t)} = \frac{E(e_t e_{t-k})}{E(e_t^2)} \quad (9.32)$$

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9.4.1 Residual Correlogram

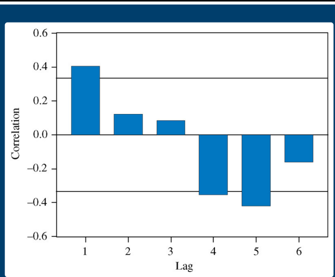

Figure 9.4 Correlogram for Least Squares Residuals from Sugar Cane Example

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9.4.1 Residual Correlogram

$$y_t = \beta_1 + \beta_2 x_t + e_t$$

$$y_t = \beta_1(1-\rho) + \beta_2 x_t + \rho y_{t-1} - \rho\beta_2 x_{t-1} + v_t$$

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9.4.1 Residual Correlogram

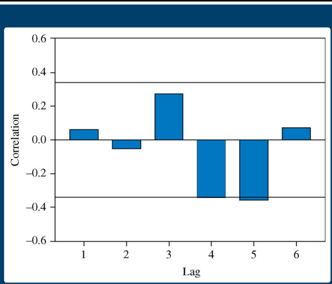

Figure 9.5 Correlogram for Nonlinear Least Squares Residuals from Sugar Cane Example

9.4.2 A Lagrange Multiplier Test

$$y_t = \beta_1 + \beta_2 x_t + \rho e_{t-1} + v_t \quad (9.33)$$

$t = 2.439$ $F = 5.949$ $p\text{-value} = .021$

$$y_t = \beta_1 + \beta_2 x_t + \rho \hat{e}_{t-1} + \hat{v}_t \quad (9.34)$$

$$b_1 + b_2 x_t + \hat{e}_t = \beta_1 + \beta_2 x_t + \rho \hat{e}_{t-1} + \hat{v}_t$$

9.4.2 A Lagrange Multiplier Test

$$\hat{e}_t = (\beta_1 - b_1) + (\beta_2 - b_2) x_t + \rho \hat{e}_{t-1} + \hat{v}_t$$
$$= \gamma_1 + \gamma_2 x_t + \rho \hat{e}_{t-1} + \hat{v}_t \quad (9.35)$$

$$LM = T \times R^2 = 34 \times .16101 = 5.474$$

Appendix 9B
The Durbin-Watson Test

$$H_0: \rho = 0 \qquad H_1: \rho > 0$$

$$d = \frac{\sum_{t=2}^{T} (\hat{e}_t - \hat{e}_{t-1})^2}{\sum_{t=1}^{T} \hat{e}_t^2} \quad (9B.1)$$

Appendix 9B
The Durbin-Watson Test

$$d = \frac{\sum_{t=2}^{T}\hat{e}_t^2 + \sum_{t=2}^{T}\hat{e}_{t-1}^2 - 2\sum_{t=2}^{T}\hat{e}_t\hat{e}_{t-1}}{\sum_{t=1}^{T}\hat{e}_t^2}$$

$$= \frac{\sum_{t=2}^{T}\hat{e}_t^2}{\sum_{t=1}^{T}\hat{e}_t^2} + \frac{\sum_{t=2}^{T}\hat{e}_{t-1}^2}{\sum_{t=1}^{T}\hat{e}_t^2} - 2\frac{\sum_{t=2}^{T}\hat{e}_t\hat{e}_{t-1}}{\sum_{t=1}^{T}\hat{e}_t^2} \qquad (9B.2)$$

$$\approx 1 + 1 - 2r_1$$

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Appendix 9B
The Durbin-Watson Test

$$d \approx 2(1 - r_1) \qquad (9B.3)$$

$$d \le d_c$$

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Appendix 9B
The Durbin-Watson Test

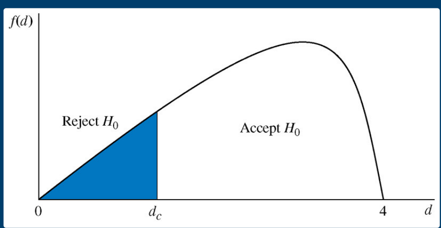

Figure 9A.1:

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Appendix 9B
9B.1 The Durbin-Watson Bounds Test

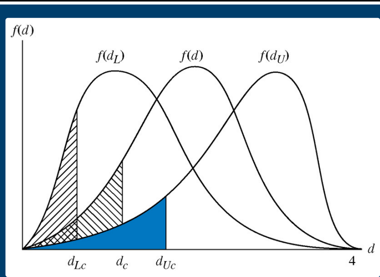

Figure 9A.2:

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Appendix 9B
9B.1 The Durbin-Watson Bounds Test

The Durbin-Watson *bounds test*.

- if $d < d_{Lc}$, reject $H_0 : \rho = 0$ and accept $H_1 : \rho > 0$;
- if $d > d_{Uc}$, do not reject $H_0 : \rho = 0$;
- if $d_{Lc} < d < d_{Uc}$, the test is inconclusive.